ON THE REDUCTION OF RESPONSE BIAS IN SAMPLE SURVEYS USING AUXILIARY INFORMATION

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SUMMARY

The control and reduction of response biases is often a major problem in sample surveys. In this paper, we develop a method for reducing response biases by using auxiliary information. When an auxiliary variable 'x' that is correlated with the variable of interest 'y' is available it is shown that the classical ratio estimator of the population mean or total of y has less response bias than the estimator that uses y - information only.

The ratio estimator, however, does not help much when the response bias for y and/or x is very large. In such situations the use of a double sampling method is useful. For a random sub-sample of the original sample either true values of y, x or values that have less biases than in the original samples are obtained. A difference estimator computed from two samples is shown to be very effective in reducing response biases.

Key Words: Response bias, ratio estimator, double sampling

1. INTRODUCTION

The general theory of sample surveys assumes that the observation y_i on the ith unit in the sample is the "true value" for that unit. The variance of an estimate obtained from the sample is assumed to arise solely from the random sampling variation that is present when only n units in the sample are measured out of the N in the population. By implication, we assume that in the case of a census (n=N) we obtain the "true value" of the mean or total of the population. In practice, however, in most surveys different types of "non-sampling errors" such as "non-response" (failure to measure some of the units in the sample), measurement error or "response error" (respondents giving in-accurate information) may be present. In this paper we are not concerned with the problem of non-response. For literature on non-response see Cochran (1977).

"Response error", in the broadest sense, means the errors that might arise from faulty measurements and observations, in-accurate answers by respondents and "interviewer bias" etc. First, we outline briefly the general theory of response errors from Madow (1965).

Let y_i be the "true value" of the character y for the ith unit of the population and y_i^* be the random variable that is the choice of the respondent i, if the respondent i is asked the question for which true value for that respondent is y_i. Let $E(y_i^*) = a_i$. Then, for the ith respondent, the response bias and the variance of the response are

$$B_i = a_i - y_i$$

and

$$V_i = E(y_i^* - a_i)^2$$

respectively. Finally, MSE of response is

$$M_i = E(y_i^* - y_i)^2 = E(y_i^* - a_i)^2 + (a_i - y_i)^2.$$

The objective of the survey is to estimate the "true population mean" $\bar{Y}$, of y and a sample of size n is drawn from the N units in the population. It can be shown that MSE of $\bar{y}^*$ (ignoring the fpc) is

$$\text{MSE}(\bar{y}^*) = \frac{S_a^2}{n} + \frac{\sigma_w^2}{n}\left[1 + (n-1)\rho_w\right] + (\bar{A}-\bar{Y})^2 \tag{1.1}$$

where

$$\sigma_w^2 = \sum_1^N \sigma_i^2/N, \quad \sigma_i^2 = \sigma_{y_i^*}^2 ,$$

$$\rho_w \sigma_w^2 = \frac{1}{N(N-1)} \sum_{i \neq j} E(y_i^* - a_i)(y_j^* - a_j)$$

and

$$\bar{A} = \sum_1^N a_i/N.$$

The formula (1.1) contains two terms S_a^2/n and $\sigma_w^2(1-\rho_w)/n$ that decrease as n increases. The remaining two terms $\sigma_w^2 \rho_w$ and $(\bar{A}-\bar{Y})^2$ are independent of n. Thus in large samples the MSE is likely to be dominated by these two terms, the ordinary sampling variance becoming unimportant and misleading as a guide to the real accuracy of the results. These results emphasize the importance of discovering and controlling response errors in sample surveys.

In recent years much of the research on sampling practice has been devoted to the study of response errors. Cochran (1977) has given an extensive discussion of this topic. Madow (1965) has suggested the use of double sampling technique using y information only for eliminating or reducing the response bias. In this paper, we develop a method for reducing response biases by using auxiliary information.

2. General Statement of the Problem

Consider a finite population of N units. Let y_i and x_i denote the true values of characteristic of interest y and auxiliary characteristic x respectively attached to the i th unit of the population. The parameter to be estimated is the population mean of y, $\bar{Y} = \sum_{i=1}^N y_i/N$. or population to total $Y = N\bar{Y}$.

From a simple random sample of n($\leq$ N) units we have the sample data (y_i^*, x_i^*), i = 1,2,...n. Note that y_i^* and x_i^* are the values reported by respondents instead of true values (y_i, x_i). The estimator of Y is

$$\hat{Y} = N\bar{y}^* \tag{2.1}$$

that uses y information only. The ratio estimator of Y is

$$\hat{Y}_r = N\,\bar{y}_r^{\,*} \qquad (2.2)$$

where $\bar{y}_r^{\,*} = (\bar{y}^{\,*} / \bar{x}^{\,*})\,\bar{X}$

and $\bar{y}^{\,*}$ and $\bar{x}^{\,*}$ are sample means of y^* x^* respectively.

We note that both estimators $\hat{Y}$ and $\hat{y}_r$ will have bias due response errors. The ratio estimators will also have the usual bias of a ratio estimator that occurs because only a fraction of the population is sampled. We shall ignore the usual bias by assuming sample size n is sufficiently large and investigate the bias due to response errors. An interesting case of response errors 'over-reporting' was found to have occured in Agricultural Surveys in Texas. To fix the idea, let us consider the agricultural surveys in Texas. After the A. S. C. S. list in each county has been consolidated, a random sample of n operator addresses will be drawn out of N addresses and data will be collected by mail questionnaire. For simplicity assume that there is 100% response to mail questionnaire. If over-reporting occurs both y_i^* and x_i^* refer to an operation which is in excess of that properly due to the i th operator of the A.S.C.S. list. Such over-reporting for certain units in the sample might be caused by the fact that the "frame" (list of units) is out of date. Over reporting can sometimes be detected by scrutiny of data and corrected by direct interview. Such a procedure is rather costly and, therefore, seldom feasible in large scale surveys. Hartley (1966) proposed the use of the ratio estimator (2.2) to eliminate the bias due to over-reporting in agricultural surveys in Texas. In this paper, we show that the ratio estimator has less response bias than the estimator that uses y-information only. The ratio estimator suggested by Hartley, however, does not help much when the response bias for y and/or x is very large. In such situations a difference estimator obtained by a double sampling method is shown to be very effective in reducing response biases.

3. Over-reporting Bias Under a Model

Conceptually, we can imagine that a large number of independent repetitions of the measurement on each unit of the population are possible. Let $y_{i\alpha}^*$ and $x_{i\alpha}^*$ be the values of the characters y and x obtained for the ith unit in the αth repetition. Then we have the model

$$y_{i\alpha}^* = y_i + \eta_{i\alpha} \qquad (3.1)$$

$$x_{i\alpha}^* = x_i + \xi_{i\alpha}$$

where, as before, y_i and x_i denote the true values of the characters y and x for the ith unit and $\eta_{i\alpha}$ and $\xi_{i\alpha}$ are errors of reporting in the αth repetition. If there is no over-reporting for the ith unit in the α th repetition then $\eta_{i\alpha} = \xi_{i\alpha} = 0$; otherwise $\eta_{i\alpha} > 0$ and $\xi_{i\alpha} > 0$. Under the repeated measurement of the units we have

$$\varepsilon\{\eta_{i\alpha} \mid y_i\} = \mu_{1i};\ \varepsilon\{\xi_{i\alpha} \mid x_i\} = \mu_{2i}$$

where ε denotes the expectation over repeated measurements.

Over-reporting bias is essentially a non-sampling error in the sense that the bias is not eliminated even in the census. In this section we, therefore, confine ourselves to the bias of the estimator $\bar{Y}_r^* = (\bar{Y}^*/\bar{X}^*)\,\bar{X}$, (rather than that of $\bar{y}_r^*$ under sampling), under the above model, to simplify the discussion.

$$\text{Now } \bar{Y}^*/\bar{X}^* = \sum_{i=1}^{N} (y_i + \eta_{i\alpha}) / \sum_{i=1}^{N} (x_i + \xi_{i\alpha})$$

$$= \frac{\bar{Y}}{\bar{X}} \left(1 + \frac{\bar{\eta}_\alpha}{\bar{Y}}\right) \left(1 + \frac{\bar{\xi}_\alpha}{\bar{X}}\right)^{-1} \qquad (3.2)$$

where $\bar{\eta}_\alpha = N^{-1} \sum_{i=1}^{N} \eta_{i\alpha}$ and $\bar{\xi}_\alpha = N^{-1} \sum_{i=1}^{N} \xi_{i\alpha}$.

Assuming $\bar{\xi}/\bar{X} < 1$, which will be generally true, and using Taylor's expansion for $(1 + \bar{\xi}_\alpha/\bar{X})^{-1}$

we get, neglecting cubic and higher order terms in $\bar{\xi}/\bar{X}$

$$\frac{\bar{Y}^*}{\bar{X}^*} \doteq \frac{\bar{Y}}{\bar{X}} \left\{ 1 + \frac{\bar{\eta}_\alpha}{\bar{Y}} - \frac{\bar{\xi}_\alpha}{\bar{X}} - \frac{\bar{\eta}_\alpha \bar{\xi}_\alpha}{\bar{Y}\,\bar{X}} + \frac{\bar{\xi}_\alpha^2}{\bar{X}^2} \right\}. \qquad (3.3)$$

Further,

$$\varepsilon(\bar{\eta}_\alpha/\bar{Y}) = \bar{\mu}_1/\bar{Y},\ \varepsilon(\bar{\xi}_\alpha/\bar{X}) = \bar{\mu}_2/\bar{X},$$

$$\varepsilon(\bar{\eta}_\alpha \bar{\xi}_\alpha/\bar{Y}\bar{X}) = \{\mathrm{Cov}(\bar{\eta}_\alpha, \bar{\xi}_\alpha) + \mu_1\mu_2\}/\bar{Y}\,\bar{X}$$

$$= \mu_1\,\mu_2\,(1 + \rho^*\,C_1\,C_2)/\,\bar{Y}\,\bar{X},$$

$$\varepsilon(\bar{\xi}_\alpha^2/\bar{X}^2) = \bar{\mu}_2^2\,(1 + C_2^2)/\,\bar{X}^2,$$

where $\bar{\mu}_1 = N^{-1}\sum_{i=1}^{N}\mu_{1i}$, $\bar{\mu}_2 = N^{-1}\sum_{i=1}^{N}\mu_{2i}$, $C_1^2 = V(\bar{\eta}_\alpha)/\bar{\mu}_1^2$,

$C_2^2 = V(\bar{\xi}_\alpha)/\bar{\mu}_2^2$ and ρ^* is the coefficient of correlation between $\bar{\eta}_\alpha$ and $\bar{\xi}_\alpha$.

Thus

$$\varepsilon\left(\frac{\bar{Y}^*}{\bar{X}^*}\right) \doteq \frac{\bar{Y}}{\bar{X}} \left\{1 + \frac{\bar{\mu}_1}{\bar{Y}} - \frac{\bar{\mu}_2}{\bar{X}} + \frac{\bar{\mu}_2^2}{\bar{X}^2}(1 + C_2^2) - \frac{\bar{\mu}_1\bar{\mu}_2}{\bar{Y}\,\bar{X}}(1 + \rho^*\,C_1C_2)\right\}. \qquad (3.4)$$

The relative bias of $\bar{Y}^*_r$ as an estimator of $\bar{Y}$ is

$$B=\bar{X}\{\varepsilon(\bar{Y}^*/\bar{X}^*)-\bar{Y}/\bar{X}\}/\bar{Y} \doteq \frac{\bar{\mu}_1}{\bar{Y}} - \frac{\bar{\mu}_2}{\bar{X}} + \frac{\bar{\mu}_2^2}{\bar{X}^2}(1+C_2^2) - \frac{\bar{\mu}_1\bar{\mu}_2}{\bar{Y}\,\bar{X}}(1+\rho^* C_1 C_2). \qquad (3.5)$$

If $\bar{\mu}_1/\bar{Y}=\bar{\mu}_2/\bar{X}$, (3.5) reduces to $B \doteq \bar{\mu}_2^2(C_2^2 - \rho^* C_1 C_2)/\bar{X}^2$.

It is of interest to investigate the magnitude of the relative bias. The relative bias clearly depends on the values of the parameters ρ^*, C_1 C_2, $\bar{\mu}_1/\bar{Y}$ and $\bar{\mu}_2/\bar{X}$. We now make the reasonable assumption

$$C_1=C_2=C \text{ (say)} \qquad (3.6)$$

to simplify the discussion and let

$$\bar{\mu}_2/\bar{X} = L\bar{\mu}_1/\bar{Y} = L\,P \quad \text{(say)} \qquad (3.7)$$

where P is the relative bias of the estimator $\bar{Y}^*$ which does not use the x-information and L is the ratio of the relative bias of $\bar{X}^*$ to that of $\bar{Y}^*$.

Then

$$B \doteq (1-L)\,(1-LP)\,P+P^2C^2\,L(L-\rho^*). \qquad (3.8)$$

We have made a numerical evaluation of the magnitude of the relative bias for different values of parameters L,P,C and ρ^*. The coefficient of variation C is of order $N^{-\frac{1}{2}}$ if the measurements on different units are uncorrelated; otherwise C could be large. We have, therefore, included small and large values of C to cover both the cases. The results are presented in Tables 1,2 and 3.

The following conslusions may be drawn from the Tables 1, 2 and 3.

(1) For P, the relative bias of the estimator $\bar{Y}^*$, not exceeding 25%, $|B|$, the relative bias of the ratio estimator $\bar{Y}^*_r$, is less than P. When P=50%, $|B| < P$ if $C \leq 1.50$. These results demonstrate the effectiveness of the ratio estimator in reducing the over-reporting bias.

(2) For fixed $L \leq 1.0$, P and C, $|B|$ decreases as ρ^* increases.

(3) For fixed L, C and ρ^*, $|B|$ increases with P.

(4) The relative bias is, for all practical purposes, negligible ($\leq$ 5%) for $.75 \leq L \leq 1.25$ and $C \leq 2.50$ if $P \leq 10\%$ even when the correlation is low ($\rho^* = .3$).

(5) For $.75 \leq L \leq 1.25$, $|B|$ is less than 6% if $P \leq 25\%$, $C \leq 1.50$ and $\rho^* > .5$.

(6) The relative bias becomes, in general, serious with P >25% and C >1.50. In such cases, it is higher for L >1 than L <1.

4. The Elimination or Reduction of Over-reporting Bias by Double Sampling

In the previous section we have shown that the ratio estimator suggested by Hartley, is generally effective in reducing the over-reporting bias. The ratio estimator, however, does not help much when the relative bias for the character 'y' and/or for the character 'x' is large. In such situations, the use of a double sampling method seems to be appropriate. In this section we, therefore, outline the double sampling technique for our present problem.

A random subsample of size n_1 (< n) out of the original sample of size n is drawn. The "true values" (y_i, x_i) are ascertained for the operators selected in the subsample either from records if that is feasible or by interviewing the selected operators. We note that true values may not always be obtained by this method but the values obtained will have smaller biases than the values (y_1^*, x_1^*) reported by the respondents. However, we suppose that the true values (y_i, x_i) are obtained for the subsample to simplify the discussion. We also assume, to simplify the discussion, that y_1^*, and x_1^* are fixed quantities, instead of random variables.

Let $\bar{y}_1^*$, $\bar{x}_1^*$, $\bar{y}_1$ and $\bar{x}_1$ be the subsample means. Then an estimator of $\bar{Y}$ is given by

$$\bar{y}'_r = t\,\bar{X} \qquad (4.1)$$

where

$$t=\frac{\bar{y}^*}{\bar{x}^*} - \left(\frac{\bar{y}_1^*}{\bar{x}_1^*} - \frac{\bar{y}_1}{\bar{x}_1}\right) \qquad (4.2)$$

is the difference estimator. Clearly the expected value of t is

$$E(t) \doteq R$$

and

$$E(\bar{y}'_r) \doteq \bar{Y}$$

provided n_1 is sufficiently large i.e., $\bar{y}'_r$ is approximately unbiased. The variance of t is given by

$$V(t)=V\left(\frac{\bar{y}^*}{\bar{x}^*}\right)+V\left(\frac{\bar{y}_1^*}{\bar{x}_1^*} - \frac{\bar{y}_1}{\bar{x}_1}\right) -2\,\mathrm{Cov}\left(\frac{\bar{y}^*}{\bar{x}^*}, \frac{\bar{y}_1^*}{\bar{x}_1^*} - \frac{\bar{y}_1}{\bar{x}_1}\right). \qquad (4.3)$$

Now, the variance of $\bar{y}^*/\bar{x}^*$ is

$$V\left(\frac{\bar{y}^*}{\bar{x}^*}\right) \doteq \frac{1}{n\,\bar{X}^{*2}}\,S^2(y^* - R^*\,x^*) \qquad (4.4)$$

(neglecting fpc), where

$$R^* = \bar{Y}^*/\bar{X}^*. \qquad (4.5)$$

Using conditional expections, it can be shown that

$$V\left(\frac{\bar{y}_1^*}{\bar{x}_1^*}-\frac{\bar{y}_1}{\bar{x}_1}\right) \doteq \frac{n-n_1}{n\,n_1}\,\frac{1}{\bar{X}^{*2}}\,S^2_{(y^*-R^*x^*)}$$

$$+\frac{1}{n\,\bar{X}^{*2}}\,S^2(y^*-R^*x^*)+\frac{n-n_1}{n\,n_1}\,\frac{1}{\bar{X}^2}\,S^2_{(y-Rx)}$$

$$+\frac{1}{n\,\bar{X}^2}\,S^2_{(y-Rx)}-\frac{n-n_1}{n\,n_1}\,\frac{2}{\bar{X}^*\bar{X}}\,S(y^*-R^*x^*)(y-Rx)$$

$$-\frac{2}{n\bar{X}^*\bar{X}}\,S_{(y^*-R^*x^*)(y-Rx)} \qquad (4.6)$$

and

$$\mathrm{Cov}\left(\frac{\bar{y}^*}{\bar{x}^*},\ \frac{\bar{y}_1^*}{\bar{x}_1^*}-\frac{\bar{y}_1}{\bar{x}_1}\right) \doteq \frac{1}{n\bar{X}^{*2}}\,S^2_{(y^*-R^*x^*)}$$

$$-\frac{1}{n\bar{X}^*\bar{X}}\,S_{(y^*-R^*x^*)(y-Rx)} \qquad (4.7)$$

Substitution of (4.4), (4.6), and (4.7) into (4.3) leads to

$$V(t)\doteq\frac{1}{n\bar{X}^2}\,S^2_{(y-Rx)}+\frac{n-n_1}{n\,n_1}\left\{\frac{1}{\bar{X}^{*2}}\,S^2_{(y^*-R^*x^*)}\right.$$

$$\left.+\frac{1}{\bar{X}^2}\,S^2_{(y-Rx)}-\frac{2}{\bar{X}^*\bar{X}}S_{(y^*-R^*x^*)(y-Rx)}\right\}$$

$$\doteq\frac{1}{n\bar{X}^2}\,S^2_{(y-Rx)}+\frac{n-n_1}{n\,n_1}\,S^2_{(r^*-r)}\ (\text{say}). \qquad (4.8)$$

Finally, we obtain the variance of $\bar{y}_r'$ as

$$V(\bar{y}_r')=\bar{X}^2\left\{\frac{1}{n\bar{X}^{*2}}\,S^2_{(y-Rx)}+\frac{n-n_1}{n\,n_1}\,S^2_{(r^*-r)}\right\}. \qquad (4.9)$$

If no subsample is selected, then from the original sample of size n the estimator of $\bar{Y}$ is $\bar{y}_r^*$ given by (4.2). The over-reporting bias of $\bar{y}_r^*$, ignoring the technical bias of the ratio estimator, is given by

$$B^*=\bar{X}(R^*-R)=\bar{Y}^*-\bar{Y} \qquad (4.10)$$

and the MSE of $\bar{y}_r^*$ is

$$\mathrm{MSE}(\bar{y}_r^*)\doteq\bar{X}^2\left\{\frac{1}{n\bar{X}^{*2}}\,S^2_{(y^*-R^*x^*)}+(R^*-R)^2\right\}. \qquad (4.11)$$

To compare the double sampling plan with single sampling we assume

$$\frac{1}{n\bar{X}^2}\,S^2_{(y-Rx)}=\frac{1}{n\bar{X}^{*2}}\,S^2(y^*-R^*x),$$

to simplify the discussion. Then from (4.9) and (4.11) we have

$$\Delta=\mathrm{MSE}(\bar{y}_r^*)-V(\bar{y}_r')=\bar{X}^2\left\{(R^*-R)^2-\frac{n-n_1}{n\,n_1}\,S^2(r^*-r)\right\}$$

$$=\left\{(\bar{Y}^*-\bar{Y})^2-\frac{n-n_1}{n\,n_1}\,\bar{X}^2\,S^2\,(r^*-r)\right\} \qquad (4.12)$$

If $B^*=\bar{Y}^*-\bar{Y}$ is small, then Δ is small or negative there is little to gain from the double sampling. As we have stated earlier, the double sampling technique will be used only when the over-reporting bias B^* is large. When B^* is large, then if $S^2(r^*-r)$ is not large compared to $(R^*-R)^2$ it would be possible with a moderate value of n_1 to make

$$\frac{n-n_1}{n\,n_1}\,\bar{X}^2\,S^2(r^*-r)<(0.1)\ (\bar{Y}^*-\bar{Y})^2;$$

even a smaller multiplier than 0.1 should not be difficult to attain. Thus the double sampling technique would be effective in reducing the bias. The costs of obtaining the true values (y_i, x_i) for the subsample may be quite high compared to the costs of collecting data by mail questionnaire for the large sample and still the double sampling technique would be efficient.

Considering appropriate cost functions for data collection by mail questionnaire and interviews and using the variance formulas for single sampling and double sampling, one could formulate an optimum double sampling scheme. We are at present working on this and hope to report the results in a subsequent paper.

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Table 1. Over-reporting Bias of the Ratio Estimator for L=.75 and Selected Values of P, C and ρ.

L = 0.75

P%	C	Bias (absolute value %)			
		ρ=.3	ρ=.5	ρ=.7	ρ=.9
10	.10	2.32	2.31	2.31	2.31
	.50	2.40	2.36	2.32	2.28
	1.00	2.65	2.50	2.35	2.20
	1.50	3.07	2.73	2.40	2.06
	2.00	3.66	3.06	2.46	1.86
	2.50	4.42	3.48	2.55	1.61
25	.10	5.10	5.09	5.08	5.07
	.50	5.61	5.37	5.14	4.90
	1.00	7.12	6.25	5.31	4.37
	1.50	9.82	7.71	5.61	3.50
	2.00	13.52	9.77	6.02	2.27
	2.50	18.26	12.40	6.54	.68
50	.10	7.90	7.86	7.82	7.78
	.50	9.92	8.98	8.05	7.11
	1.00	16.25	12.50	8.75	5.00
	1.50	26.80	18.36	9.99	1.48
	2.00	41.56	26.56	11.56	3.44
	2.50	60.55	37.11	13.67	9.77

Table 2. Over-reporting Bias of the Ratio Estimator for L=1.00 and Selected Values of P, C and ρ.

L = 1.00

P%	C	Bias (absolute value %)			
		ρ=.3	ρ=.5	ρ=.7	ρ=.9
10	.10	.007	.005	.003	.001
	.50	.17	.12	.07	.02
	1.00	.70	.50	.30	.10
	1.50	1.58	1.12	.67	.22
	2.00	2.80	2.00	1.20	.40
	2.50	4.38	3.12	1.87	.62
25	.10	.04	.03	.02	.01
	.50	1.09	.78	.47	.16
	1.00	4.37	3.12	1.87	.62
	1.50	9.84	7.03	4.22	1.41
	2.00	17.50	12.50	7.50	2.50
	2.50	27.34	19.53	11.72	3.91
50	.10	.17	.12	.07	.02
	.50	4.37	3.12	1.87	.62
	1.00	17.50	12.50	7.50	2.50
	1.50	39.37	28.12	16.87	5.62
	2.00	70.00	50.00	30.00	10.00
	2.50	109.37	78.12	46.87	15.62

Table 3. Over-reporting Bias of the Ratio Estimator for L=1.25 and Selected Values of P, C and ρ.

L = 1.25

P%	C	Bias (absolute value %)			
		ρ=.3	ρ=.5	ρ=.7	ρ=.9
10	.10	2.18	2.18	2.18	2.18
	.50	1.89	1.95	2.01	2.07
	1.00	1.00	1.25	1.50	1.75
	1.50	.48	.08	.64	1.20
	2.00	2.56	1.56	.56	.43
	2.50	5.23	3.67	2.11	.55
25	.10	4.22	4.24	4.25	4.26
	.50	2.44	2.83	3.22	3.61
	1.00	3.12	1.56	0.00	1.56
	1.50	12.40	8.89	5.37	1.86
	2.00	25.39	19.14	12.89	6.64
	2.50	42.08	32.32	22.56	12.79
50	.10	4.39	4.45	4.52	4.57
	.50	2.73	1.17	.39	1.95
	1.00	25.00	18.75	12.50	6.25
	1.50	62.11	48.05	33.98	19.92
	2.00	114.06	87.06	64.06	39.06
	2.50	180.86	141.80	102.73	63.67